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## **Sediment Transport**

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*This course was adapted from the U.S. Department of Agriculture. National Resources Conservation Service. Part 631 Geology National Engineering Handbook, Chapter 23 “Sediment Transport”, which is in the public domain.*

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## **Factors Affecting Sediment Transport**

### **Introduction**

The individual characteristics of water and sediment and their interaction directly affect the type and volume of material eroded and transported and the place and rate of deposition. Evaluating channel instability, including erosion or aggradation, and predicting the performance of proposed changes are problems that require knowledge of the sediment transport processes. The amount and kind of sediment transported in a project affect the overall project planning, design, and maintenance and replacement requirements.

This course includes a discussion of the characteristics of water as a medium for initiating the movement and transport of sediment. The reaction of material on the streambed to the hydraulic forces exerted and the effects of velocity and flow depth on the rate of bed-material transport are described. Formulas and procedures designed to predict the rate of bed-material transport are reviewed. Terms and symbols are described in the last section, 'Symbols'.

### **Characteristics of Water as the Transporting Medium**

The interrelated characteristics of water that affect its ability to entrain and move sedimentary particles are density, viscosity, and chemistry.

Density is the ratio of mass to volume. Increasing the temperature of water increases its volume and decreases its density. With an increase in temperature from 40° to 100° C (104° to 212° F), water expands to 1.04 times its original volume. In working with large volumes of moving water, the slight variations in density that result from temperature change are usually ignored.

Viscosity is the cohesive force between particles of a fluid that causes the fluid to resist a relative sliding motion of particles. Under ordinary pressure, viscosity varies only with temperature. A decrease in water temperature from 26.7° to 4.4° C (80° to 40° F) increases viscosity by about 80 percent. Changes in viscosity affect the fall velocity of suspended sediment and its vertical distribution in turbulent flow (Colby and Scott 1965). Increasing the viscosity lowers the fall velocity of particles, particularly very fine sands and silts. The combination of substantial decrease in water temperature and the consequent increase in viscosity result in smoothing of the bed configuration, lowering the Manning "n" roughness coefficient, and increasing the velocity over a sand bed (U.S. Department of the Army 1968).

In acid waters sediment deposition may be promoted by the formation of colloidal masses of very fine sediments (flocculation) that settle faster than their component fine particles. The pH value is the negative logarithm (base 10) of the hydrogen ion concentration. Neutral water has a pH value of 7.0. Acid water has a pH value lower than 7.0; alkaline water has a pH value higher than 7.0.

### **Laminar Sublayer**

In turbulent flow, a thin laminar sublayer forms adjacent to the bed. The flow is laminar because the fluid particles in contact with the bed do not move. The higher the velocity or the lower the viscosity, the thinner is this sublayer. If the boundary is rough enough, its irregularities may project into the theoretical laminar sublayer and prevent its actual development.

Although laminar flow is primarily related to fluid viscosity, turbulent flow is affected by a number of factors. In laminar flow, filaments of water follow parallel paths; but in turbulent flow, the paths of particles crisscross and touch, mixing the liquid. A criterion defining the transition from laminar to turbulent flow is the Reynolds number,  $R_e$ , a ratio of inertial force to shear force on the fluid particle. If the Reynolds number is low, shear forces are dominant, but as the Reynolds number increases, they decline to little significance, thereby indicating the dominance of inertial forces.

The association of laminar flow with viscosity and that of turbulent flow with inertia are the same whether the fluid is moving or at rest. A small particle of sediment, such as very fine sand, settling in still or flowing water, moves slowly enough to sustain laminar flow lines in relatively viscous media. Inertial forces become increasingly important as grain size increases. Inertial forces are dominant when the particle size exceeds 0.5 mm.

### **Characteristics of Transportable Materials**

The entrainment and transport of granular materials depend on the size, shape, and specific weight of the particles and their relative positions. The resistance of cohesive materials depends largely on the forces of interparticle bonding. Cohesive forces can be attributed to several factors, including the amount and kind of clay minerals, the degree of consolidation or cementation, and the structure of the soil mass.

### **Sediment Transport in Fluvial Systems**

Table 1 illustrates how the total sediment load is classified, based on mechanism of transport and by particle size:

- a. bedload
- b. bed-material load
- c. wash load

Reports may also use generic terms such as sediment yield, sediment load, or sediment production. Understanding the type of sediment and the forces that mobilize and move it are critical for designing systems that are stable and perform efficiently.

Table 1. Sediment load classification.

		Classification System	
		Based on Mechanism of Transport	Based on Particle Size
<b>Total sediment load</b>	Wash load	Suspended load	Wash load
	Suspended bed-material load		Bed-material load
	Bedload	Bedload	

Adapted from Cooper and Peterson 1970



## Mechanism of Entrainment

### Forces Acting on Discrete Particles

Turbulence is a highly irregular motion characterized by the presence of eddies. The degree to which eddies form depends on the boundary roughness and geometry of the channel. Eddies penetrate the laminar sublayer formed along the bed.

Discrete particles resting on the bed are acted on by two components of the forces associated with the flow. One component of force is exerted parallel to the flow (drag force) and the other is perpendicular to the flow (lifting force). Drag force results from the difference in pressure between the front and the back sides of a particle. Lifting force results from the difference in pressure on the upper and lower surfaces. If the lifting force exceeds the particle's immersed weight and the interference of neighboring grains, the particle goes into suspension.

Because turbulence is random and irregular, discrete particles tend to move in a series of short, intermittent bursts. In each burst, particles move a short distance, and many grains move simultaneously. Then the movement subsides until another burst occurs. The frequency and extent of movement increases with the intensity of turbulence. Above a certain intensity, some particles may be projected into the flow as suspended load (Sutherland 1967). The coarser and rounder the particles, however, the greater the possibility that they will begin to roll and continue rolling.

### Tractive Force

The instantaneous interactions between turbulent flow and discrete sediment particles resting on the bed were described briefly in the preceding paragraphs. In practical application, however, it is more convenient to use time-average values of the force field generated by the flow near the bed. Here, the forces normal to the bed having a time average equal to zero can be eliminated, and only those forces parallel to the bed need to be considered. The time average of these forces is the tractive force. The tractive force measured over a unit surface area is the tractive stress. In a prismatic channel reach of uniform flow bounded by two end sections, the mean value of tractive stress is equal to the weight of the water prism in the reach multiplied by the energy gradient and divided by the wetted boundary surface in the reach. Shear stress or force per unit area of bed is expressed as  $\tau_0 = \gamma R S_e$ . Terms and symbols are described in final section of this course, 'Symbols'.

### Determining Critical Tractive Stress

The most widely used and most reliable evaluation of tractive stress related to the initiation of motion is that developed by Shields (1936). The theoretical concepts, supported by experiments, resulted in a plot of

$$\frac{\tau_c}{\gamma \left( \frac{\gamma_s}{\gamma} - 1 \right) d_s} \quad \text{against} \quad \frac{U_* d_s}{\nu} \quad .$$

The first expression is an entrainment function and the second is the boundary Reynolds number, indicating the intensity of flow turbulence around the particle. The Shields data are based on particles of uniform size and a flat bed. The Shields experiments indicate that beyond a certain value of the boundary Reynolds number,  $\frac{U_* d_s}{\nu}$ , the value of the parameter  $\frac{\tau_c}{\gamma \left( \frac{\gamma_s}{\gamma} - 1 \right) d_s}$  remains constant.

Within these limits, the critical tractive stress is therefore proportional to grain size. The critical tractive stress is the force that initiates mobilization.

Data on critical tractive stresses obtained in a number of investigations were assembled by Lane (1955). These data show that the critical tractive stress in pounds per square foot is equal to  $\tau_c = 0.5d_{75}$ , where  $d_{75}$  is the size in inches of the bank material at which 25 percent by weight is larger. The limiting (allowable) tractive stress was determined from observations of canals (Lane 1955). The recommended limiting tractive stress in pounds per square foot is equal to 0.4 of the  $d_{75}$  size in inches, for particles that exceed 0.25 in diameter. Results of experiments on finer particles vary considerably.

In figure 1 the critical tractive (shear) stress is plotted against the mean particle size or the  $d_{75}$ . The figure shows the differences in critical tractive stress resulting from temperature variation and the boundary Reynolds number at various tractive stress levels. The wide departure of Lane's curve for critical tractive stress from the others in is believed to be due to Lane's use of the data of Fortier and Scobey (1926) from canals after aging. The stability of some soils is increased by aging. Figure 1 is from Shields (1936), Lane (1955), and American Society of Civil Engineers (1975, p. 99).

### Determining Critical Velocity

Determining critical velocity (the velocity at which particles in the bed begin to move) is another method for establishing stability criteria. Figure 2 shows critical water velocity as a function of mean grain size (American Society of Civil Engineers 1975). There has been less agreement on critical velocity than on critical tractive stress, probably because bottom velocity increases more slowly with increasing depth than does mean velocity. Critical conditions for initiating movement can be expressed directly in terms of tractive stress, but critical mean velocity must be related to variation in velocity with depth.

Determining the correct critical value for tractive stress or velocity is important when designing a threshold channel, where the boundaries are designed not to erode. The significance of the critical value is determined by the magnitude and duration of flows that initiate sediment movement. A prolonged flow slightly exceeding the critical value may have little effect on bed material transported. A brief flow substantially exceeding the critical value, however, could transport a large volume of sediment.

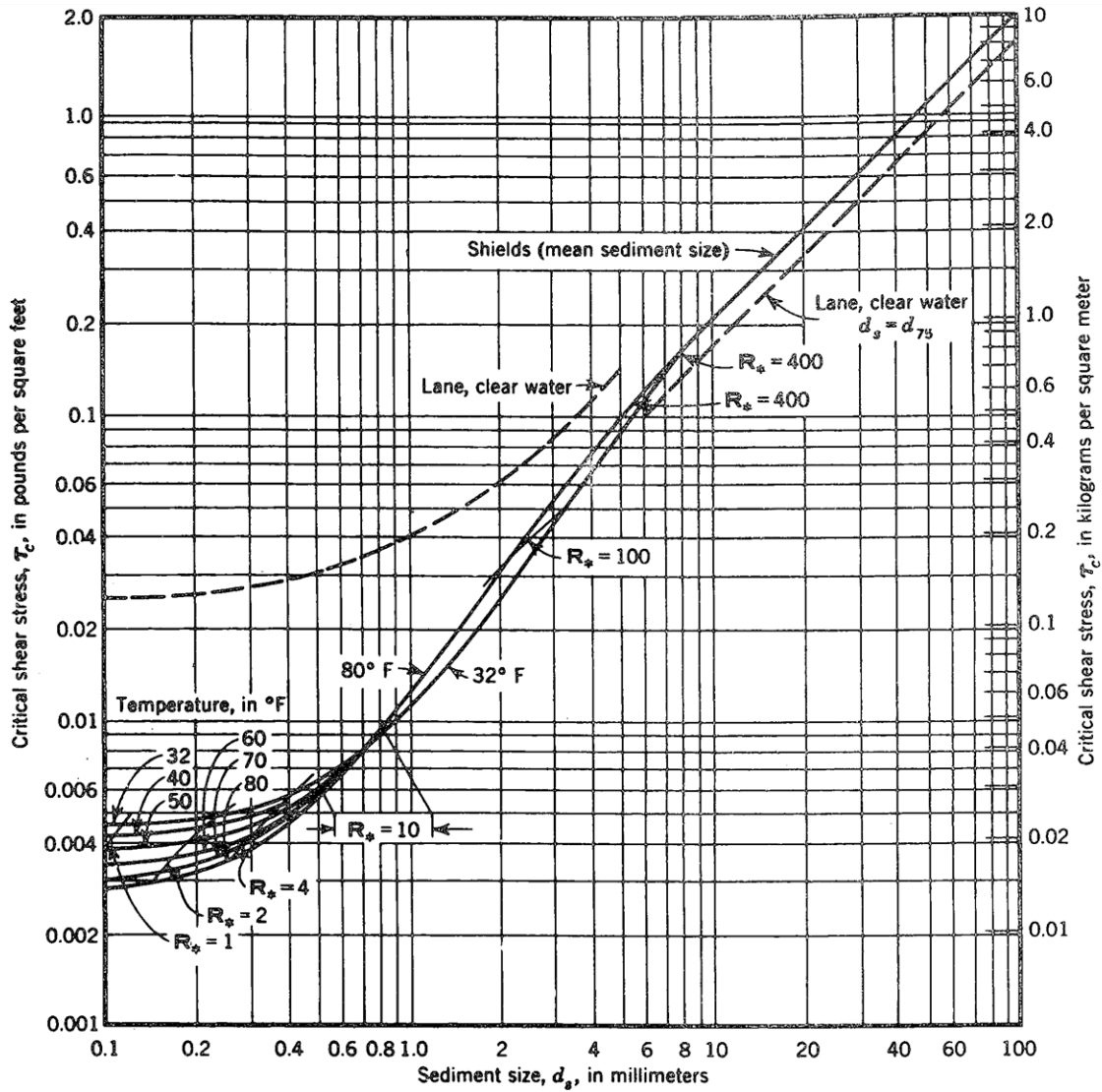


Figure 1. Critical shear stress for quartz sediment in water as a function of grain size.

American Society of Civil Engineers 1975.

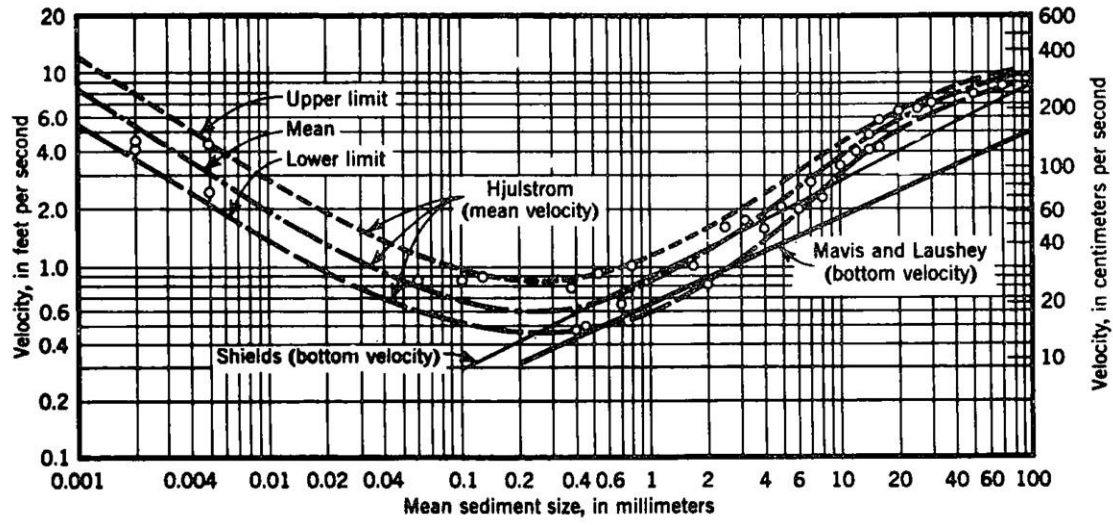


Figure 2. Critical water velocity for quartz sediment as a function of mean grain size.

## Hydraulic Considerations

### Fixed Boundaries

The relationships of velocity, stage, and discharge for stream channels with fixed boundaries have long been satisfactorily predicted by selecting the appropriate “n” value in Manning’s and other related formulas.

### Movable Boundaries

Ripples, ripples on dunes, or dunes may form at a low transport rate, and antidunes or a flatbed may form at a high transport rate. These bed forms have been observed in sand-bed flumes and in streams with a  $d_{50}$  size finer than 1.0 mm. The variety of bed forms in coarser material is smaller.

For analyses, the hydraulic radius is divided into two parts (Einstein 1950; Einstein and Barbarossa 1952):

- a. The radius resulting from the roughness of the grain size of the individual particles ( $R'$ ).
- b. The radius resulting from the roughness of the bed configuration ( $R''$ ).

From field observations, Einstein and Barbarossa developed a graph relating the dimensionless ratio

$$\frac{V}{U_*''} \quad (\text{where } U_*'' = (gR''S_e)^{1/2}) \text{ to Einstein's flow-intensity parameter, } \psi.$$

This graph indicates that for a given set of conditions, it is possible to develop a unique stage-discharge relationship and to predict the hydraulics of a channel with movable boundaries. Vanoni and Brooks (1957) presented a graphical solution to the friction equation from which ( $R'$ ) is determined.

Another procedure for predicting hydraulic behavior in movable channel beds is based on the division of slope,  $S$ , into two parts,  $S'$  and  $S''$  (Meyer-Peter and Muller 1948). In this procedure  $S'$  is the energy gradient associated with the grain size of the bed material under a certain velocity and depth, excluding form resistance, and  $S''$  is the additional gradient pertaining to bed-form resistance. A similar hydraulic consideration sometimes used as part of the preliminary procedure in transport computations is the treatment of bank friction as completely distinct from bed friction. One such approach, involving use of Manning’s friction equation, is included as part of the procedure in the Einstein bedload function.

## **Suspended Sediment**

### **Transport of Suspended Sediments**

Suspended-sediment load includes both the bed-material load in suspension and the wash load, as shown in table 1. Erosion of fine-textured soils usually produces wash load and not the bed-material load for the bulk of the sediment discharge. Predicting rates of wash-load transport is done using stream gage data showing concentrations of suspended sediment during measured discharges.

### **Settling Rate**

The settling rate for sediment particles of uniform density increases with size, but not proportionally. The settling rate for particles smaller than about 0.062 mm varies approximately as the square of the particle diameter, whereas particles of coarse sand settle at a rate that varies approximately as the square root of the diameter. The settling rate for particles of intermediate size varies at an intermediate rate.

The lateral distribution of suspended sediment across a stream is fairly uniform in both deep and shallow flows except below the junction of a tributary carrying material at a concentration substantially different from that of the mainstream. The flow from the tributary tends to remain on the entrance side of the channel for some distance downstream, as shown in figure 3.



*Figure 3. Sediment-laden water entering stream from tributary.*

The dividing line between sediments classed as silts and those classed as sands is the 0.074-mm size. Clay and silt particles usually are distributed fairly uniformly in stream, but sand particles usually are more concentrated near the bottom. The degree of variation is a function of the coarseness of the particle (figure 4)

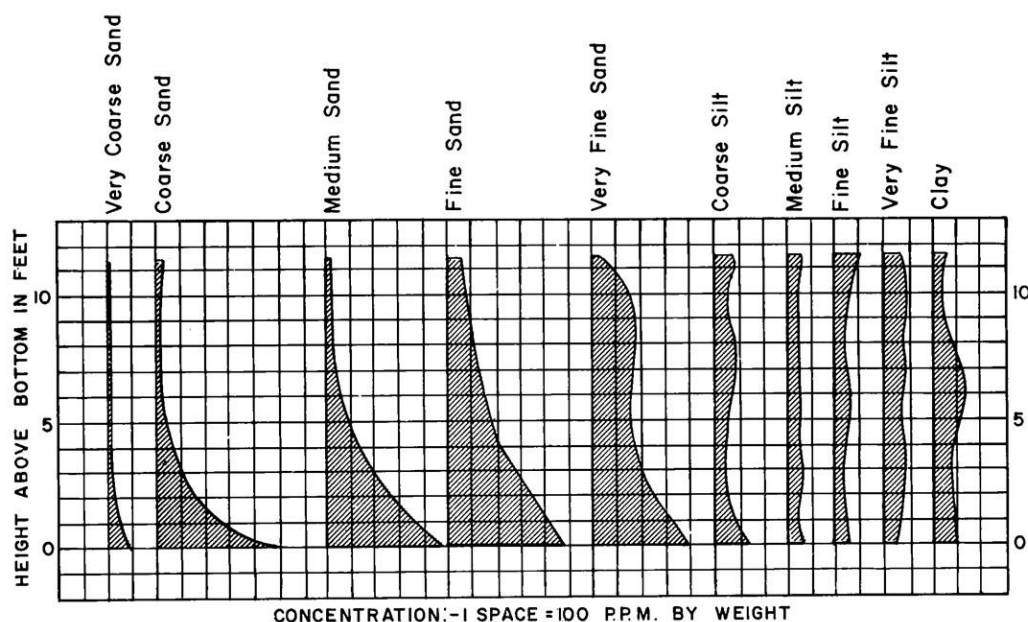


Figure 4. Vertical distribution of sediment in Missouri River at Kansas City, MO.

From Federal Inter-Agency River Basin Committee (1963, p. 28)

### Sampling and Laboratory Procedures

The Technical Committee (TC), Federal Interagency Sedimentation Project (FISP), was formerly sponsored directly by member agencies of the Advisory Committee on Water Information's (ACWI) Subcommittee on Sedimentation (SOS). The Technical Committee has standardized the design, manufacturing, and calibration of isokinetic sediment samplers.

The U.S. Geological Survey collects most of the suspended-sediment samples in the U.S. Samples are collected by lowering and raising a depth-integrating sampler that samples the flow at a uniform isokinetic rate. Collecting a valid sediment sample requires that the stream flow be measured at the time of sampling. An isokinetic sampler collects a water-sediment sample from the stream at a rate such that the velocity in the intake nozzle is equal to the incident stream velocity at the nozzle entrance. The water-sediment sample collected is proportional to the instantaneous stream velocity at the locus of the intake nozzle, and therefore is representative of the sediment load at that point (Davies 2005). Travel time to and from the streambed is regulated so that the container is not quite full of the water-sediment mixture when it returns to the surface.

New sampling techniques include the use of acoustic Doppler technology and laser devices for recording rates of sediment transport directly, without securing samples. Such devices allow rapid and continuous measurement of sediment loads, with proper use and calibration (Gray and Gartner 2009).

Research and field tests have shown that turbidity readings cannot be directly correlated with actual suspended sediment loads because of the effect of incidence of light (angle of reflection), coloration of suspended sediment particles, shape of particles, presence of algae, staining agents (as in tannins), and the subjectivity of light reflection to the one taking the turbidity measurements.

Laboratory procedures used in handling the samples include weighing the container holding the water-sediment mixture and then decanting the clear liquid, evaporating the remaining moisture, and weighing the dry sediment. The ratio of the dry weight of the sediment times  $10^6$  to the weight of the water-sediment mixture is the sediment concentration in parts per million. The suspended-sediment concentration can be expressed in milligrams per liter by using equation 1. (American Society of Civil Engineers 1975, p. 403). Factor A is given in table 23-6.

$$\text{Concentration in milligrams/liter} = A \left( \frac{\text{weight of sediment} \times 10^6}{\text{weight of water and sediment mixture}} \right) \quad \text{Eq. 1.}$$

Table 2. Factor A for computing sediment concentration in milligrams per liter by equation 1.

<b><math>\frac{\text{Wt. of sediment}}{\text{Wt. of sediment and water}} \times 10^6</math></b>			
<b>Concentration</b>	<b>A</b>	<b>Concentration</b>	<b>A</b>
0 – 15,900	1.00	322,000 – 341,000	1.26
16,000 – 46,900	1.02	342,000 – 361,000	1.28
47,000 – 76,900	1.04	362,000 – 380,000	1.30
77,000 – 105,000	1.06	381,000 – 398,000	1.32
106,000 – 132,000	1.08	399,000 – 416,000	1.34
133,000 – 159,000	1.10	417,000 – 434,000	1.36
160,000 – 184,000	1.12	435,000 – 451,000	1.38
185,000 – 209,000	1.14	452,000 – 467,000	1.40
210,000 – 233,000	1.16	468,000 – 483,000	1.42
234,000 – 256,000	1.18	484,000 – 498,000	1.44
257,000 – 279,000	1.20	499,000 – 513,000	1.46
280,000 – 300,000	1.22	514,000 – 528,000	1.48
301,000 – 321,000	1.24	529,000 – 542,000	1.50



### Sediment-Rating Curve and Flow-Duration Curve Method of Computing Suspended-Sediment Load

Periodic data on suspended sediment or short-term daily data are sometimes extended for use as average annual yields by constructing sediment-transport and flow-duration curves, or what has become known as sediment rating curves. A sediment rating curve constructed by plotting discharge and sediment-load data in tons is shown in figure 5.

To construct a flow-duration curve, divide data on mean discharges into a series of classes over a range that has been recorded at this station. Then count the number of days within each class. Determine the percentage of time in each class and plot the midpoint on log-probability paper against the accumulated percentage at that point. Table 3 is an example of a flow-duration curve.

Figure 6 illustrates how to use the sediment-transport curve and the flow-duration curve to determine the annual sediment yield for the period of record. Each segment of the curve represents the proportion of a composite day in which a particular flow occurs during the period of record. For example, in table 3 discharge is 100 ft<sup>3</sup>/s or greater for 10 percent of a composite day. Methods of preparing flow-duration curves are described in detail by Miller (1951) and Searcy (1959).

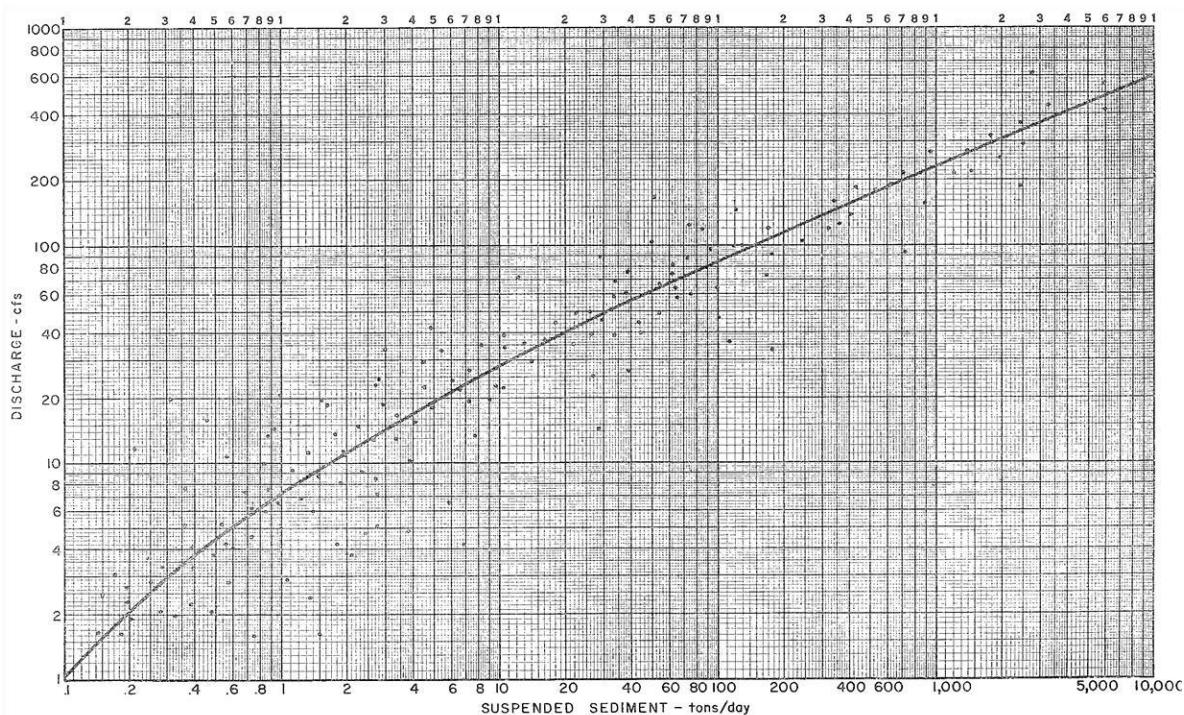


Figure 5. Sediment rating curve, any Creek, any State.

Table 3. Computation of average annual suspended-sediment load, any Creek, any State.

1	2	3	4	5	6	7
Percentage limits	Percentage interval	Percentage (mid ordinate)	Water Discharge $Q_w$	Sediment load, $Q_s$	Water Discharge ( $Q_w$ ) per day Col. 2 x Col. 4	Sediment load ( $Q_s$ ) per day Col. 2 x Col. 5
(%)	(%)	(%)	(ft <sup>3</sup> /sec)	(tons)	(ft <sup>3</sup> /sec)	(tons/day)
0.01 – 0.05	0.04	0.030	590	9,000	0.24	3.6
0.05 – 0.1	0.05	0.075	505	6,400	0.25	3.2
0.1 – 0.5	0.4	0.30	400	3,500	1.6	14.0
0.5 – 1.5	1.0	1.0	310	1,900	3.1	19.0
1.5 – 5	3.5	3.25	200	700	7.0	24.5
5 – 15	10	10	100	145	10.0	14.5
15 – 25	10	20	47	28	4.7	2.8
25 – 35	10	30	25	8	2.5	0.8
35 – 45	10	40	13	3	1.3	0.3
45 – 55	10	50	7	1	0.7	0.1
55 – 65	10	60	4	0.5	0.4	0.05
65 – 75	10	70	3	—	0.3	—
75 – 85	10	80	2	—	0.2	—
85 – 95	10	90	1	—	0.1	—
Total					32.39	82.8

Annual sediment load =  $82.8 \times 365.25 = 30,240$  tons per year

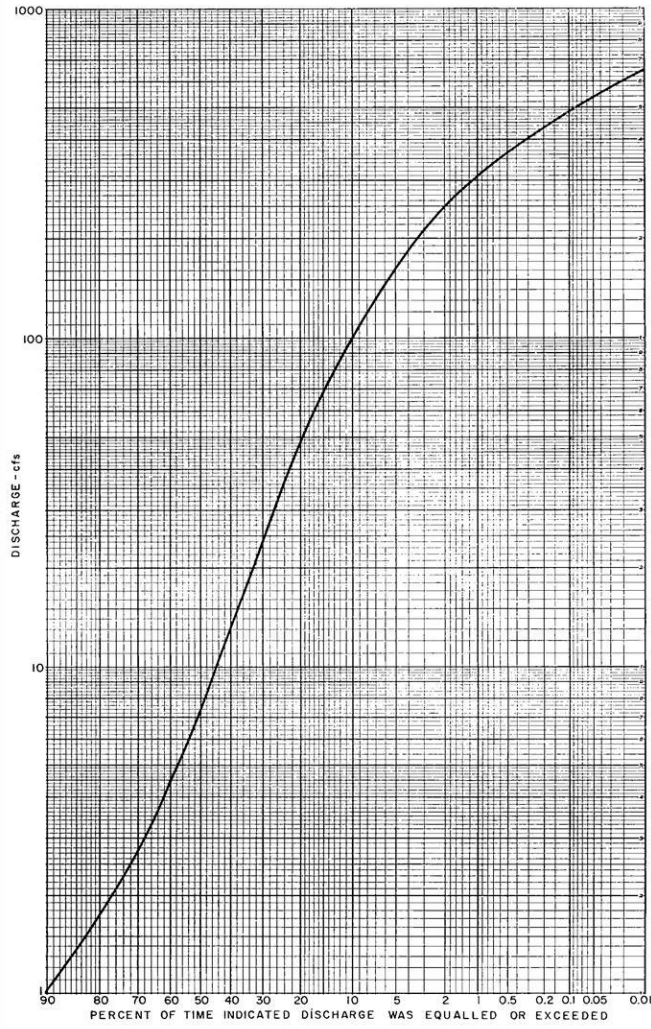


Figure 6. Flow-duration curve, any Creek, any State.

## Transport of Bed Material

### Bed Material Transport

Bed-material load is defined as the part of the total sediment load (suspended load plus bedload) that is composed of grain sizes occurring in appreciable quantities in the bed material. The part of the total load that consists of grain sizes that are not present in significant quantities in the bed material is the wash load. Sand-size particles that constitute all or the major part of the bed material travel either on the bed as bedload or in suspension. Transport rates for sand and gravel are determined by both direct measurement and through computation.

The earliest bed-material transport formula still in use is that of DuBoys, who published results of studies of the Rhone River in 1879 (DuBoys 1879). DuBoys originated a concept common to many later formulas when he assumed in his derivation that the rate of sediment transport is proportional to the tractive stress in excess of the critical value required to initiate motion.

The DuBoys formula is:

$$Q_T = \psi \tau_0 (\tau_0 - \tau_c) \quad \text{Eq. 2.}$$

where:

$q_T$  = rate of sediment transport per unit width of stream;

$\psi$  = a coefficient that depends on characteristics of the sediment (not to be confused with Einstein's  $\psi$ );  $\tau_c$  = a value established by experiment (not the same as that of Shields).

Early in the twentieth century, several flume studies of sand transport were started, including that of Shields. He is best known for developing criteria for the initiation of movement.

### Einstein Bedload Function and Bed Material Transport

The Einstein bedload function, the Engelund-Hansen procedure, and the Colby procedure determine the rate of bed-material transport, both as bedload and suspended load. Bedload transport formulas were developed by Schoklitsch, Meyer-Peter, Haywood, and Meyer-Peter and Muller. In 1950 Einstein's bedload function had a major effect on investigations of the hydraulics and sediment transport characteristics of alluvial streams. Einstein (1950) described the function as "giving rates at which flows of any magnitude in a given channel will transport as bedload the individual sediment sizes of which the channel bed is composed." It was developed based on experimental data, theory of turbulent flow, field data, and intuitive concepts of sediment transport.

The figures in column 1, table 3, refer to segments of the flow-duration curve. For example, the entries in horizontal line 1 are for the segment between 0.01 percent and 0.05 percent of the composite day. The Einstein bedload function first computes bedload and then, by integrating the concentration at the bed layer with the normal reflection of that concentration in the remainder of the flow depth, determines the total bed-material load.

Einstein introduced several new ideas into the theory of sediment transport. Included were new methods of accounting for bed friction by dividing it into two parts: one pertaining to the sand-grain surface and the other to the bed-form roughness, such as ripples or dunes. An additional friction factor, that of the banks, is included in the procedure for determining hydraulic behavior before computing bed-material transport.

Another idea introduced by Einstein to explain the bedload function is that the statistical properties of turbulence govern the transport of particles as bedload. This statistical character is reflected in the structure of the dimensionless parameter  $\phi$ , defined as the intensity of bedload transport. The relationship between this factor and the dimensionless flow intensity,  $\psi$  (another dimensionless parameter reflecting the intensity of shear on the particle) is used in the procedure. The  $\phi - \psi$  relationship has subsequently been tested by others and found to be an appropriate determinant of bedload transport.

### Engelund-Hansen Procedure for Bed Material Transport

Engelund and Hansen (1967) developed a procedure for predicting stage-discharge relationships and sediment transport in alluvial streams. They introduced a parameter  $\theta$  (the reciprocal of Einstein's  $\psi$ ) to represent the ratio of agitating forces (horizontal drag and lifting force) to the stabilizing force (immersed weight of the particle). This parameter is a dimensionless form of the bed shear,  $\tau_0$  to be divided into two parts:  $\tau'$ , the part acting directly as traction on the particle surface, and  $\tau''$ , the residual part corresponding to bed-form drag. This division is similar to that of the Einstein-Barbarossa  $R'$  and  $R''$ . The authors' diagram of the relationship of bed forms to the two separations of total bed shear and to velocity is shown in figure 7. Principles of hydraulic similarity were used to develop a working hypothesis for describing total resistance to flow, specifically for dune-covered streambeds and bed-material discharge.

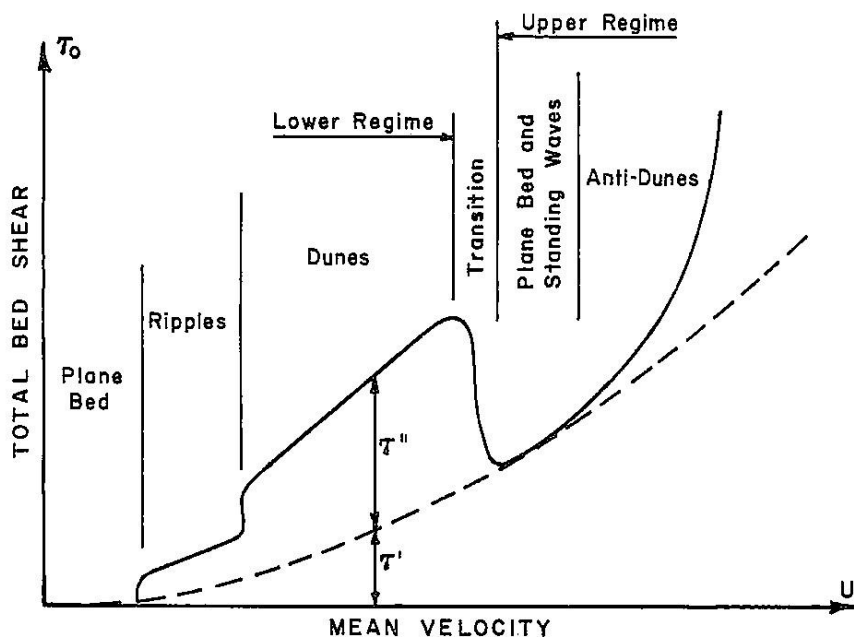


Figure 7. Relationship between grain roughness ( $\tau'$ ) and form drag ( $\tau''$ ) and total bed shear ( $\tau_0$ ).  
From Engelund and Hansen (1967).

The steps used in applying the Engelund-Hansen procedure are given here in some detail because the procedure demonstrates the impact of changing bed forms on bed material transport and because it was published in a foreign journal not readily available for reference. Data from flume experiments (Guy, Simons, and Richardson 1966) were used to test the Engelund-Hansen theories. The mean sizes used in these experiments were 0.19, 0.27, 0.45, and 0.93 mm. Transport of the bed material, both in suspension and moving along the bed, was measured.

The Engelund-Hansen procedure includes both a simplified and a more detailed series of computations. Figure 8 in conjunction with table 3 shows the flow regime in which a semi graphical solution, figure 7, applies; that is, in the region of dune formation. The formula is as follows in equation 3. An online solution application is available at <https://ponce.sdsu.edu/onlineengelundhansen.php>.

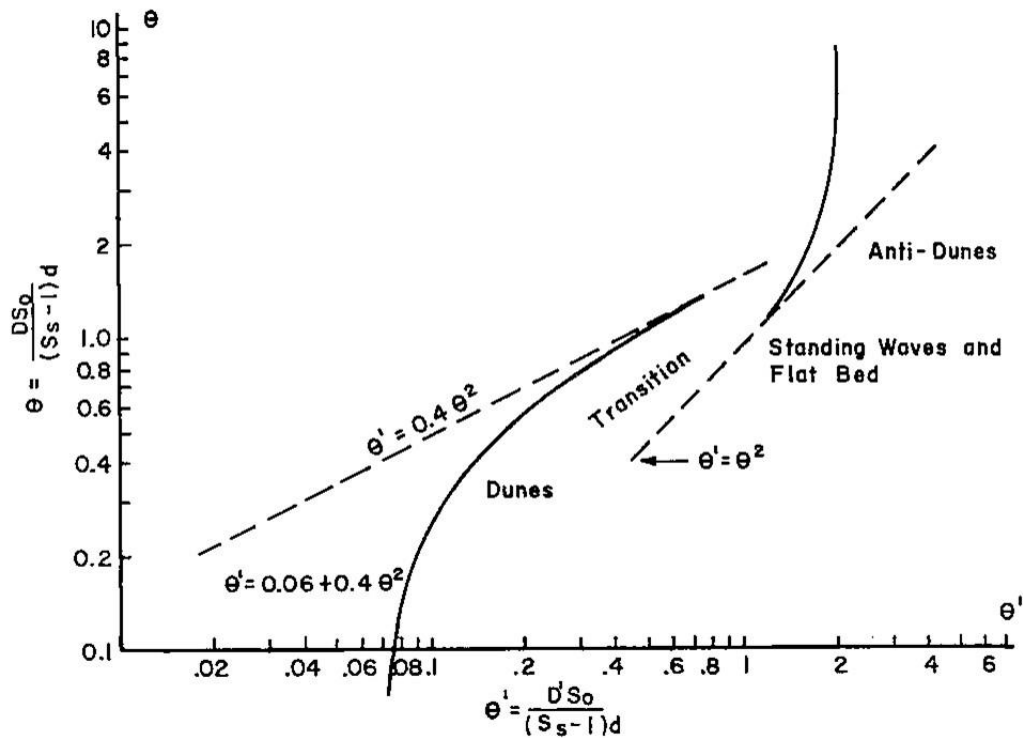


Figure 8. Relationship between dimensionless forms of bed shear ( $\theta$  and  $\theta'$ ).

From Engelund and Hansen (1967) and American Society of Civil Engineers (1975, p. 135).

$$g_s = V^2 \left( \frac{\tau_b}{(\gamma_s - \gamma) d_{50}} \right)^{\frac{3}{2}} \sqrt{\frac{d_{50}}{g \left( \frac{\gamma_s}{\gamma} - 1 \right)}} = V^2 (\tau^*)^{\frac{3}{2}} \sqrt{\frac{d_{50}}{g \left( \frac{\gamma_s}{\gamma} - 1 \right)}} \quad \text{Eq. 3}$$

where:

$g_s$  = Sediment transport by unit width

$\gamma$  = Unit weight of water  
 $\gamma_s$  = Unit weight of sediment  
 $V$  = Average channel velocity  
 $\tau_b$  = Bed shear stress  
 $\tau^*$  = Dimensionless Shields Number ( $\tau_b/(\gamma_s \gamma_d 50)$ )

The steps in applying the graphical form are as follow:

Example 1 (using the authors' symbols)

given:

$D = 1.219 \text{ m}$   
 $d = \text{mean fall diameter (m)} = 3.2 \times 10^{-4} \text{ m}$   
 $S_0 = \text{slope of the channel (m/m)} = 2.17 \times 10^{-4}$   
 $S_s = \text{specific gravity of sediment} = 2.68$

Calculate the ratio of the mean depth in meters,  $D$ , to the mean particle size fall diameter in meters,  $d$ , of the bed material.

$$\frac{D}{d} = \frac{1.219}{3.2 \times 10^{-4}} = 3.81 \times 10^3 \quad \text{Eq. 4}$$

where:

$S_0$  (figure 9) =  $2.17 \times 10^{-4}$

$$\left[ \frac{q}{(S_s - 1)gd^3} \right]^{1/2} = 3.3 \times 10^4 \text{ and } \phi = 1.5$$

then:

$$\begin{aligned}
 q &= [(S_s - 1)gd^3]^{1/2} (3.3 \times 10^4) \\
 &= [1.68(9.8)(3.2 \times 10^{-4})^3]^{1/2} (3.3 \times 10^4) \\
 &= 0.766 \text{ m}^3/(\text{s} \cdot \text{m}) = 8.25 \text{ ft}^3/(\text{s} \cdot \text{ft})
 \end{aligned}$$

and:

$$\begin{aligned}
 q_T &= \phi [(S_s - 1)gd^3]^{1/2} \\
 &= 1.5 [1.68(9.8)(3.2 \times 10^{-4})^3]^{1/2} \\
 &= 3.48 \times 10^{-5} \text{ m}^3/(\text{s} \cdot \text{m}) \\
 &= 0.000375 \text{ ft}^3/(\text{s} \cdot \text{m})
 \end{aligned}$$

At  $95 \text{ lb/ft}^3$ , sediment by weight is:  
 $= 95 \times 0.000375 = 0.036 \text{ lb}/(\text{s} \cdot \text{ft})$

Example 2 shows early in the computation that the long form of computations must be followed.

Example 2

given:

$D = \text{mean depth of } 1.0 \text{ ft} = 0.3048 \text{ m}$

$d$  = mean fall diameter of  $3.2 \times 10^{-4}$  m  
 $S_s = 2.68$   
 $S_0$  = slope of the channel = 0.002

$$\frac{D}{d} = \frac{0.3048}{3.2 \times 10^{-4}} = 9.52 \times 10^2 \quad \text{Eq. 5.}$$

These values fall to the right of the lined chart (figure 8) and probably within the transition and *plane-bed* regime.

$$\theta = \frac{DS_0}{(S_s-1)d} = \frac{(0.3048)(0.002)}{(1.68)(0.00032)} \quad \text{Eq. 6.}$$

(See figures 7 and 8)

$$\theta = 1.134$$

where:

$\theta'$  = for transition or plane bed regime  
 $= 0.4 \theta^2 = 0.514$

$$D' = \text{boundary layer thickness} = \frac{\theta'}{\theta} D = \frac{0.514}{1.134} (0.3048)$$

where:

$k$  = surface roughness as determined by Engelund-Hansen  
 $= 2.5d = 2.5(0.32) = 0.80$  mm

$$\frac{U}{[gD'S_0]^{1/2}} = 6.0 + 5.75 \log \frac{D'}{k} \quad \text{in millimeters}$$

$$U = [9.8(0.138)(0.002)]^{1/2} \times \left[ 6.0 + 5.75 \log \frac{138}{0.80} \right] = 0.98 \text{ m/s}$$

$$U = 3.22 \text{ ft/s}$$

$$\therefore \text{discharge} = 3.22 \text{ ft}^3/(\text{s} \cdot \text{ft})$$

The bed-material discharge can be calculated as follows:

$$f\phi = 0.1 \theta^{5/2} \quad \text{Eq. 7.}$$

(as determined by Engelund-Hansen)

where:

$$\begin{aligned}
 f &= \text{friction factor} = \frac{2gS_0D}{U^2} \\
 &= \frac{2(9.8)(0.002)(0.3048)}{(0.981)^2} = 0.0124
 \end{aligned}$$

then:



$$\phi = \frac{0.1}{f} \theta^{5/2} = \frac{0.1}{0.0124} 1.134^{5/2} = 11.04$$

and:

$$q_T = \phi[(S_s - 1)gd^3]^{1/2}$$

$$= 11.04[1.68 \times 9.8(3.2 \times 10^{-4})^3]^{1/2}$$

$$= 2.564 \times 10^{-4} \text{ m}^3/(\text{s}\cdot\text{m}) = 2.76 \times 10^{-3} \text{ ft}/(\text{s}\cdot\text{ft})$$

At 95 lb/ft<sup>3</sup>, sediment by weight is 0.262 lb/(s·ft).

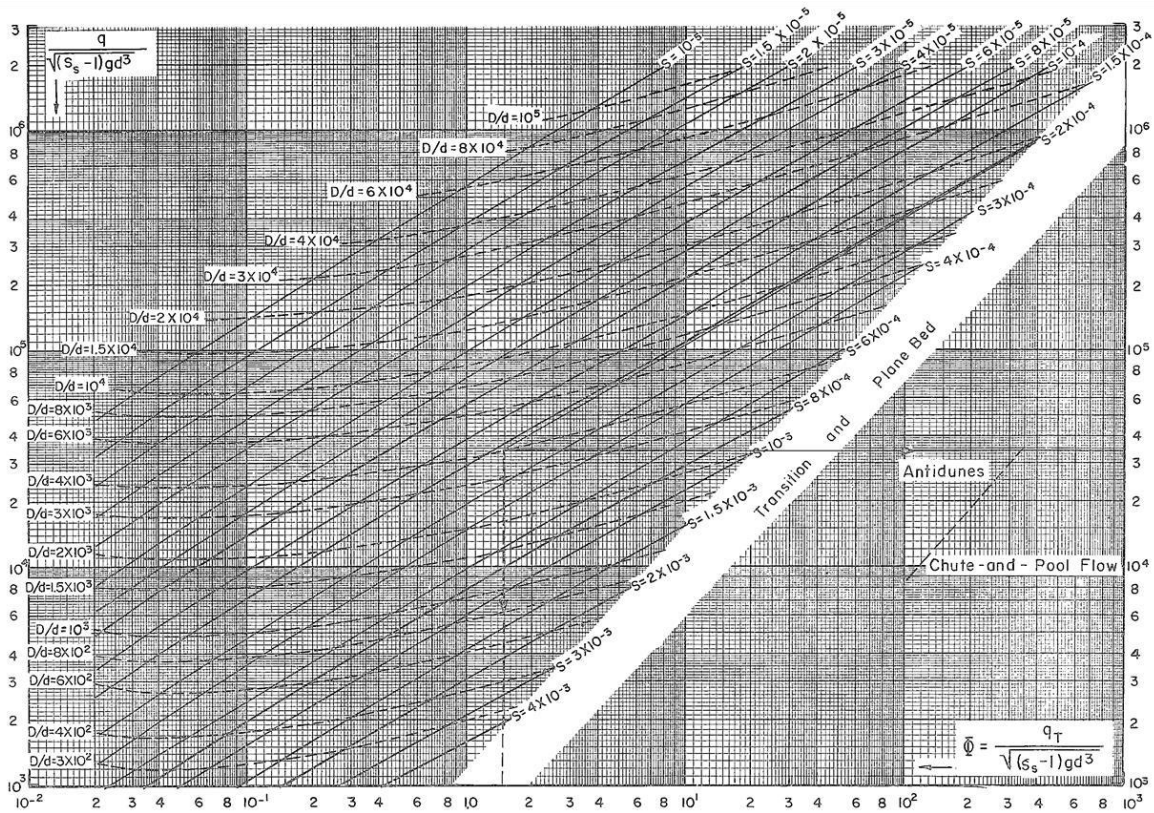


Figure 9. Graphical solution to Eq. 6,  $\frac{q}{\sqrt{(S_s - 1)gd^3}}$  and  $\phi$  in the Engelund-Hansen procedure.

Adapted from Engelund and Hansen (1967) and American Society of Civil Engineers (1975, p. 209)

In summary, the velocity of 3.22 ft/s, discharge of 3.22 ft<sup>3</sup>/(s·ft), and bed-material transport of 0.262 lb/(s·ft) are determined for a transitional or upper plane-bed regime. The Engelund-Hansen procedure does not provide a means for determining the bed-material discharge at lower flow regimes of plane beds and ripples. These regimes are not significant, however, in terms of the volume of sediment transported.

### Colby Procedure for Relating Mean Velocity to Bed Material Transport

The Colby procedure was developed by correlating mean velocity with sediment concentration in a sand-bed stream (Colby 1964). The procedure, partly empirical and partly derived from Einstein’s bedload function, is based on measurements in flumes and channels. The relationships are presented in table 3,

which gives the uncorrected sand transport as a function of velocity, depth, and the  $d_{50}$  particle size of bed material for water depths (D) of 0.1, 1, 10, and 100 feet. Each of the four sets contains curves corresponding to  $d_{50} = 0.10, 0.20, 0.30, 0.40, 0.60,$  and  $0.80$  mm.

Before the graphs in figure 10 can be used, velocity must be determined by observation or calculation. The bed-material load for flows with a depth, other than the four values for which curves are given, can be determined by reading the sand transport per foot of width ( $q_T$ ) for the known velocity for the two depths indicated in figure 10 that bracket the desired depth. A log-log plot of D versus  $q_T$  enables interpolation of the bed-material load for the desired depth.

This bed-material load corresponds to a water temperature of 60°F and to material with negligible amounts of fine particles in suspension. The two correction factors,  $K_1$  and  $K_2$ , in figure 11a compensate for the effect of water temperature and concentration of fine suspended sediment on sediment discharge if the  $d_{50}$  size of bed sediment is about 0.2 to 0.3 mm. Figure 11b represents an estimate of the relative effect of concentration of fine sediment or of water temperature for  $d_{50}$  sizes of bed sediment different from those in figure 11a. For sizes other than 0.2 and 0.3 mm, multiply the adjustment coefficients from figure 11a minus 1.00 by the percentages from figure 11b. For example, if an adjustment coefficient ( $K_1$  or  $K_2$ ) from the main diagram is 1.50 and the  $d_{50}$  size of the bed sediment is 0.5 mm, then  $K_3$  from figure 11b is 60 percent of 0.5 or 30 percent. The final adjustment coefficient would be 1.30. Colby emphasized that only rough estimates can be derived from figure 11.

### Using the Colby Graphs to Determine the Discharge of Sands

The discharge of sands in a sand-bed stream can be computed from the graphs as follows:

#### Example 3

Determine sand discharge from figure 10.

given:

$$\begin{aligned}\text{Mean velocity} &= 5.8 \text{ ft/s} \\ \text{Depth} &= 8.5 \text{ ft} \\ d_{50} \text{ size of bed sediments} &= 0.26 \text{ mm}\end{aligned}$$

Figure 10 shows that discharges of sands for the given  $d_{50}$  size are about 80 and 180 tons/(day·ft) for depths of 1 and 10 ft, respectively. Interpolation for the depth of 8.5 ft on a log-log plot indicates a bed-material discharge of 170 tons per day per foot of width. No corrections are required for temperature, concentration, or sediment size; therefore, the answer is 170 tons.

#### Example 4

Determine discharge of sand from figures 10, 11a, and 11b.

given:

$$\begin{aligned}\text{Mean velocity} &= 5.8 \text{ ft/s} \\ \text{Depth} &= 8.5 \text{ ft} \\ d_{50} \text{ size of bed sediments} &= 0.60 \text{ mm}\end{aligned}$$

Water temperature = 75 °F  
Concentration of fine bed sediment = 20,000 ppm

From figure 10, the indicated discharges of sands for the given size of 0.60 mm are about 70 and 110 tons/(day·ft) for depths of 1 and 10 ft, respectively. Interpolation indicates a sand load of 105 tons per day per foot of width for a depth of 8.5 ft. The adjustment coefficient for 75° F ( $K_1$ ) on figure 11 is 0.85 and that for a fine suspended-load concentration of 20,000 ppm ( $K_2$ ) is 1.55. According to figure 11, the effect of sediment size is only 40 percent as great for a diameter of 0.60 mm as it is for a diameter of 0.20 or 0.30 mm. Therefore, 40 percent of  $(1.55 - 1.00) = 0.22$ . The value 0.22 is then added to 1.00 to obtain the estimated adjustment coefficient for a diameter of 0.60 mm. The 105 tons/(day ft) multiplied by 0.85 and by 1.22 gives 109 tons per day per foot of width.

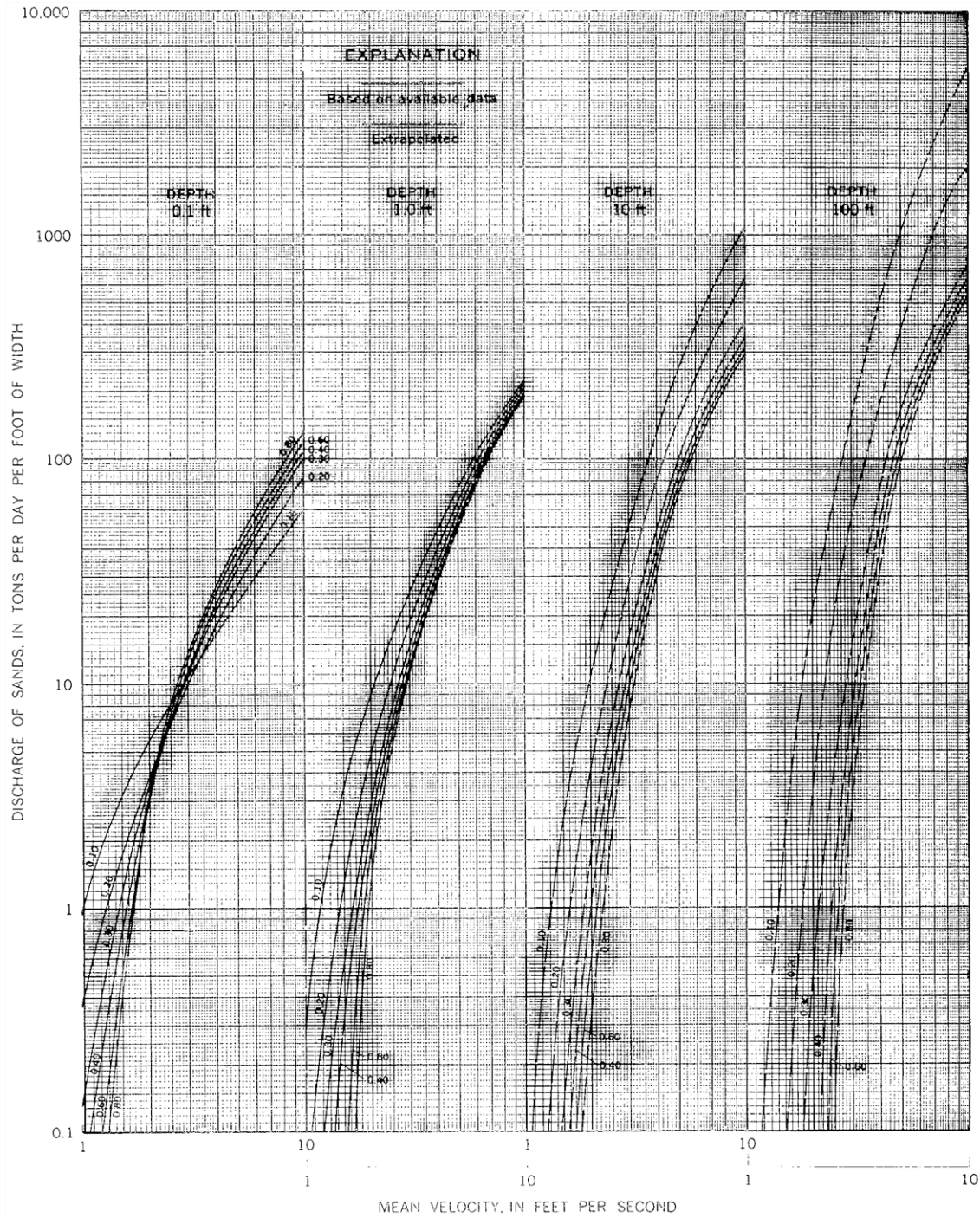


Figure 10. Relationship of discharge of sands to mean velocity for six median sizes of bed sand, four depths of flow, and a water temperature of 60° F.

From Colby (1964) and American Society of Civil Engineers (1975, p. 204).

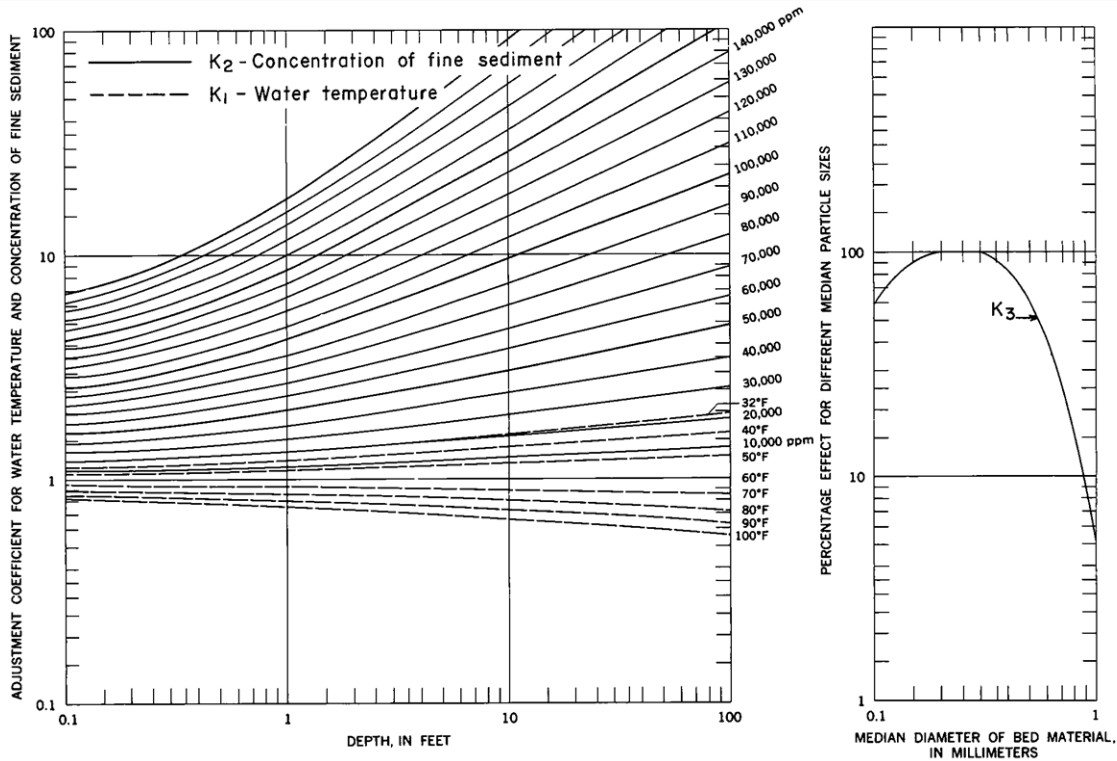


Figure 11. Approximate correction factors for the effect of water temperature and concentration of fine sediment (11a) and sediment size (11b) on the relationship of discharge of sands to mean velocity.

From Colby (1964) and American Society of Civil Engineers (1975, p. 205).

### Bedload Transport

#### Schoklitsch Bedload Transport Formula

Schoklitsch developed one of the more extensively used empirical formulas (Shulits 1935; Shulits and Hill 1968). He used his own experimental data and also data from Gilbert’s flume measurements (Gilbert 1914). The 1934 Schoklitsch formula in English units is:

$$q_B = \frac{86.7}{(d_{50})^{1/2}} S_e^{3/2} (q - q_0) \tag{Eq. 8}$$

where:

- $q_B$  = unit bedload discharge (pounds per second per foot of width);
- $d_{50}$  = medium size of sediment (inches);

$$q_0 = 0.00532 \frac{d_{50}}{S_0^{4/3}}$$

In describing the formula, Shulits recommended using a cross section in a straight reach of river where the depth of water is as uniform as possible, and the width changes as little as possible with stage (Shulits

1935). As described by Shulits, the Schoklitsch formula fits Gilbert's measurements for uniform particle sizes of about 0.3 to 7 mm and slopes ranging from 0.006 to 0.030 ft/ft for small particles, and 0.004 to 0.028 ft/ft for larger particles.

#### *Meyer-Peter Bedload Transport Formula*

In 1934 the Laboratory for Hydraulic Research at Zurich, Switzerland, published a bedload transport formula based on flume experiments with material of uniform grain size. The original analysis of the Zurich and Gilbert data for uniform particles ranging from about 3 to 28 mm in diameter was supplemented by studies of mixtures of various-sized particles up to 10 mm and having various specific gravities. The Meyer-Peter formula in English units is:

$$q_B = (39.25 q^{2/3} S_0 - 9.95 d_m)^{3/2} \quad \text{Eq. 9.}$$

Where  $d_m$  is expressed in feet. The new term in this formula is  $d_m$ , the effective diameter of the bed material, which identifies the characteristic size of a sample. To determine this value, divide the size distribution curve of a bed-material mechanical analysis into at least 10 equal size fractions and determine the mean size and weight percentage of each fraction.

#### *Haywood Bedload Transport Formula*

Haywood assumed that the discharge effective in moving bedload is midway between the discharge of walls offering no resistance and that of walls offering the same resistance as the bed (Haywood 1940). Haywood demonstrated the close relationship of his formula to the Schoklitsch formula, which is based on some of the same data. Haywood believed that his formula substantially agrees with Schoklitsch's formula for relatively large rates of bedload movement, and that it is much more accurate for very small rates of movement.

Haywood considered 3 mm to be the maximum particle size for application of his formula. He regarded his formula as a modification of the Meyer-Peter formula. The Haywood formula is:

$$q_B = \left( \frac{q^{2/3} S_0 - 1.20 d^{4/3}}{0.117 d^{1/3}} \right) \quad \text{Eq. 10.}$$

Where  $d$  is  $d_{35}$  expressed in feet.

#### *Meyer-Peter and Muller Bedload Transport Formula*

The Meyer-Peter and Muller formula is based on data obtained from continuing the experiments that resulted in the Meyer-Peter formula. The range of variables, particularly slope, was extended. A few tests were run with slopes as steep as 20 percent and sediment sizes as coarse as 30 mm.

Meyer-Peter and Muller stated explicitly that their work was on bedload transport, by which they meant the movement of sediment that rolls or jumps along the bed.

Transport of material in suspension is not included (Meyer-Peter and Muller, 1948). The Meyer-Peter and Muller formula as translated by Sheppard (1960) is:

$$q_B = 1.606 \left[ 3.306 \left( \frac{Q_s}{Q} \right) \left( \frac{d_{90}^{1/6}}{n_s} \right)^{3/2} DS_e^{-0.627} d_m \right]^{3/2} \quad \text{Eq. 11.}$$

Where  $d_{90}$  and  $d_m$  are expressed in millimeters.

Nomographs are available for determining  $Q_s/Q$  (a ratio of the discharge quantity determining bedload transport to the total discharge) and  $n_s$  (a Manning "n" value for the streambed). The formula, a significant departure from the previously cited formulas, includes a ratio of the form roughness of the bed to the grain roughness of the bed surface.

## **Application and Limitations of Formulas**

### **Sediment Transport Problems**

The lack of certainty in solving specific sediment-transport problems is in part a result of the extremely limited number of situations in which predictive techniques, such as bedload or bed-material transport formulas, have been substantiated by field measurement. See table 4 for characteristics of bed material transport and flow characteristics versus sand and gravel supply variability.

Figure 12 illustrates a few of the major factors that can be considered in the application and limitations of sediment transport formulas. The availability of bed material ranges from no sand (box A) to an unlimited supply of sand in sizes less than 1 mm (box C), to bed material of gravel and boulders (box E). Flow characteristics range from highly unsteady or rapidly changing to steady and slowly changing. Of the possible conditions illustrated by this diagram, the condition in box 2C most nearly fits the flow and sediment conditions used in developing transport formulas. Box 1C pertains specifically to smaller streams for which NRCS provides assistance, not to rivers in which deep steady flows may transport gravel as they do sand. Through limited reaches and during high flows, shallow streams may also transport gravel and boulders. Frequently a transition from scour to deposition occurs over a relatively short reach. Boxes adjacent to 2C (1C, 2B, 2D) can be considered a “gray” area for which correct solutions to sediment transport problems can be obtained by including the appropriate modifiers, such as changes in slope, to match variations in discharge.

### **Aggradation**

Aggradation occurs in some channels, even though hydraulic computations indicate that sediment should not deposit. It is not always known whether the aggradation occurred in the rising or falling stage of the hydrograph. Some of the unpredicted changes can be explained by variable bed roughness, not accounted for in conventional hydraulic computations. Variable bed roughness does not necessarily explain all the inaccuracies in predicting the effects of hydraulic change on sediment transport because some procedures do take into account the changes in bed roughness with various flows. Part of the problem may be due to unsteady flow, since steady-flow procedures fail to account for differences between stage and discharge.

Conditions favoring bed-material transport at or near a constant and predictable rate do not include delivery in slurries or other forms that change the viscosity and natural sorting processes of flow. Alluvial fills of mountain or foothill canyons are typical of conditions favoring viscous flow. Heavy storm runoff after many years of fill accumulation may produce debris or mud flows whose volume can be predicted only by field measurement. On the basis of these considerations, the treatments shown in table 4 are suggested for sediment problems in streams as categorized in figure 12.



Table 4. Bed Material Transport Considerations.

<b>1A, 2A</b>	For cohesive soil, cemented gravel, and rock, initiation of movement is the important factor in channel scour or bank erosion. Critical tractive force is related to the $d_{75}$ of bank materials. Undisturbed cohesive soil exhibits erosion resistance that may result from one or several characteristics such as structure, permeability, consolidation, cementation, or cohesion. The influence of each of these characteristics has not been identified. Their cumulative effect on erosion resistance, however, can be determined by shear strength tests on undisturbed soil that has been saturated to duplicate moisture conditions during channel flow (Flaxman 1963).
<b>1B, 2B</b>	A bed only partially covered with sand and exposing different material (cohesive soil, rock, etc.) as the fixed channel boundary indicates a limited sand supply at this specific location. Sediment transport formulas applied to this condition usually yield computed rates that exceed the actual rate. Test the potential for bank erosion by tractive force theory if the bank is composed of noncohesive materials; otherwise, use the procedures for cohesive soils.
<b>1C, 2C</b>	A sand-covered bed is the condition used in sediment transport formulas if the problem to be solved requires (a) estimating the volume of bed-material transport during a specific interval of time and at a specific level of discharge or (b) comparing the bed-material transport in a reach with that in another reach in which changes in slope, cross section, or discharge may influence the design of a channel. If flow is unsteady, replace the steady-state procedures with the proper unsteady flow relationships, as previously mentioned.
<b>2D</b>	Techniques for predicting transport rates of sand-gravel mixtures allow estimates of the potential for scour or aggradation. The probable depth of scour can be estimated by determining whether the maximum tractive force for a given flow will exceed the critical for the coarsest 5 to 10 percent of bed material. If the maximum tractive force exceeds the critical for the $d_{90}$ to $d_{95}$ , the depth of scour cannot be predicted unless still coarser material underlies the bed surface material. The amount of scour necessary to develop armor formed of the coarsest fraction can be determined from either the depth of scour or the volume of material removed in reaching this depth.
<b>1D, 1E, 2E</b>	For gravel and gravel-boulder mixtures, the technique used for determining depth of scour and volume of material produced by scour is similar to that for sand-gravel mixtures (2D). Do not use bedload formulas for this type of material unless confined flow, steepness of slope, and uniformity of cross section provide relatively uniform discharge per foot of width. The highly variable velocity and discharge per foot of width in many alluvial channels is particularly conducive to deposition alternating with scour of coarse bed material.

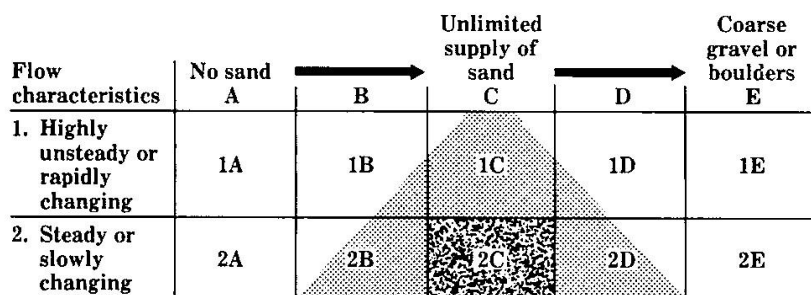


Figure 12. Characteristics of bed material transport.

## **Summary of Procedures for Evaluating Bed-Material Transport Problems**

### **Bed-Material Transport Problems**

Problems of bed-material transport require consideration of three elements:

- a. Existing conditions
- b. Availability of bed material
- c. Natural or artificial changes in stream or watershed conditions

The existing conditions can be best determined by field investigation and analysis. Surveys of old and new cross sections, use of techniques for identifying depth of scour or aggradation, and comparison of aerial photographs all facilitate definition of the problems. Although the correct identification and analysis of existing bed-material transport conditions are important, most problems require projections of what will or can occur, rather than what is now occurring. The availability of bed material and the impact of change are the key elements of such projections.

Equilibrium can be achieved only if the quantity of bed material that is being introduced into the reach is comparable to quantity of bed material moving out of the reach. Problems arise when the amount introduced is greater or less than the transport capacity of the flow. In other words, equilibrium transport seldom causes problems, but a change from equilibrium to nonequilibrium transport often does.

The supply of bed material can exceed transport capacity during unusually high discharges. This excess can be caused by development of new and substantial sources of bed material within or adjacent to the problem reach, or by channel changes that may increase transport capacity in the upstream reach but not in the downstream reach. Determining the availability of bed material is largely a field problem. To be readily available to channel flow, sediment must be in the stream system. The coarse particles in an upland soil tend to lag behind during erosion.

Gullies that feed directly into the stream system and that expose soils with a large proportion of particles of bed-material size can be major contributors but do not in themselves constitute an immediate and unlimited stream channel supply.

### **Stream Bed Supply Considerations**

Streambanks that have soil textures comparable to those in the bed can be a ready source of supply, depending on the erodibility of the material. A frequently used emergency flood-protection measure is to bulldoze streambed materials to each side to form banks or levees. These banks are a ready source of supply. Bank erosion and the consequent deterioration of channel alignment may result in overloading the flow and cause downstream aggradation.

Scour of bed material can result from an under-supply of sediment in an alluvial reach. Upstream changes in watershed or stream conditions that can reduce the supply of incoming bed material include the removal

of supply by major flood scour and the construction of reservoirs, debris basins, or other structures. Increases in flow conditions can also cause scour, due to water diversions or other increases in water discharged to the system.

In addition to cutting off the supply of bed material to the reach downstream, a reservoir can materially influence the stability of the channel bed and banks by modifying the flow. For example, a detention structure that controls a high flood peak can extend the duration of released flows by days. The resulting bed and bank scour may be extensive because of the extended duration of flows and the lack of sediment in the water.

Table 5 is a checklist of procedures to consider in solving problems of bed material transport. The last column in this table indicates that a field evaluation is important to the solution of any such problem. Because of the variety of factors that can influence their solution, most problems are not routine and solving them requires the assistance of well-trained and experienced personnel. The first step should always be a field evaluation of existing or potential problems related to sediment transport.

- a. If formulas must be used, it should be recognized that the results are qualitative and not quantitative.
- b. Observations of similar streams having comparable drainage areas, geology, soils, topography, and runoff often provide guidance on the probable stability.

Table 5. Checklist of procedures for solving bed-material transport problems.

Item	Analysis procedure			
	Tractive stress <sup>1/</sup>	Comparative hydraulics <sup>2/</sup>	Bed material formulas	Field evaluation
Erodibility of bed	✓			✓
Erodibility of bed and banks	✓			✓
Erodibility of banks	✓			✓
Channel aggradation		✓	✓	
Volume of bed material			✓	✓
Effects of channel change		✓	✓	✓
<b>Channel boundary characteristics</b>				✓
Cohesive soils	✓			✓
Cohesive soils or rock with intermittent deposits of sand or gravel	✓			✓
Sand <1.0 mm	✓	✓	✓	✓
Sand <1.0 mm with <10% gravel	✓	✓	✓	✓
Gravel, gravel mixed with sand	✓		✓	✓
Gravel and boulders	✓		✓	✓
<b>Hydraulic characteristics: In problem reach:<sup>3/</sup></b>				
Steady state or slowly changing	✓	✓	✓	✓
Rapidly changing	✓	✓		✓
<b>Cross section—slope upstream vs problem reach:</b>				
About the same	✓	✓	✓	✓
Steeper slope	✓	✓	✓	✓
Wider channel	✓	✓	✓	✓
Narrower channel	✓	✓	✓	✓

<sup>1/</sup>For cohesive soil boundaries, analysis may include tractive power (tractive stress times mean velocity).<sup>2/</sup>Comparison of relationships between depth, velocity, and unit discharge in two or more reaches.<sup>3/</sup>Special situations, see figure 12.

## **Comparison of Predictive Methods**

### **Predicted Transport Rates**

Figures 13 to 15 compare the measured and predicted transport rates of bed material sediment. The predicted rates were computed by a number of formulas, except that the total bed-material discharge for the Colorado River at Taylor's Ferry (figure 15) was determined from suspended-sediment samples by using the modified Einstein method (U.S. Department of the Interior 1958).

The formula-derived transport rates of bed-material sediment in Mountain Creek (figure 13) follow the general trend of measurements more closely than the comparable rates for the Niobrara and Colorado Rivers (figures 14 and 15, respectively). The transport characteristics of Mountain Creek may be more like the flume conditions from which most formulas were derived than like the transport conditions for the two rivers.

### **Measured Transport Rates**

In an analysis in Sedimentation Engineering (American Society for Civil Engineers 1975), measurements in figures 14 and 15 were compared with rates computed by several formulas. It was concluded that calculated curves with slopes almost the same as those fitting the data (measurements) are useful even if they do not give the correct values of sediment discharge.

The Colby procedure was derived in part from the Niobrara River data, and that the close correspondence between the measured rates and the computed rates could be expected for this reason. The Meyer-Peter or Meyer-Peter and Muller bedload formulas may be applicable for gravel and gravel-boulder mixtures with the limitations for 1D, 1E, and 2E. Appropriate formulas should be used only to relate transport capacity between one reach and another and do not yield dependable quantitative results.

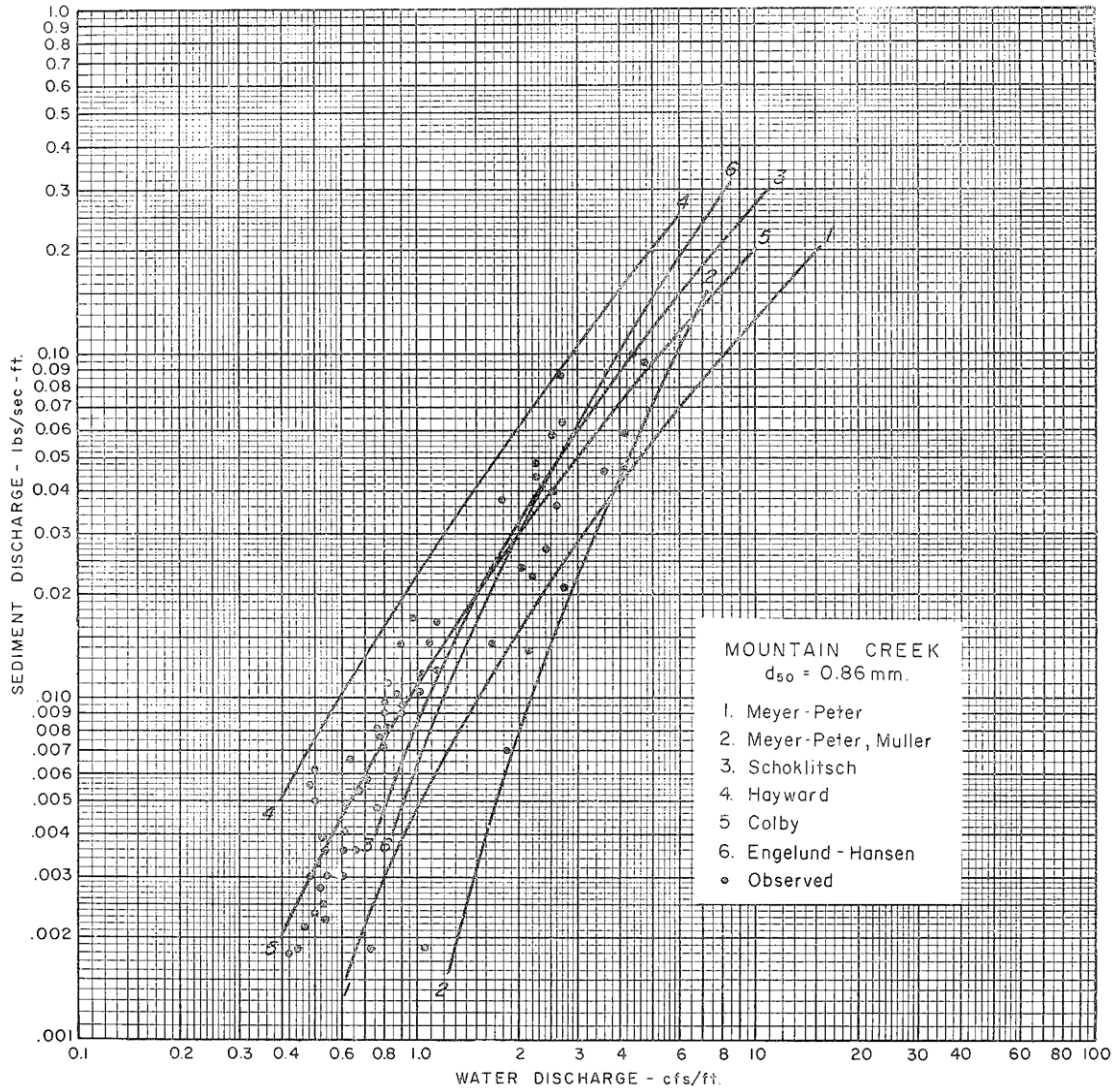


Figure 13. Sediment rating curves for Mountain Creek near Greenville, SC, according to several formulas compared with measurements.

Adapted from Vanoni, Brooks, and Kennedy (1961, p. 7-8).

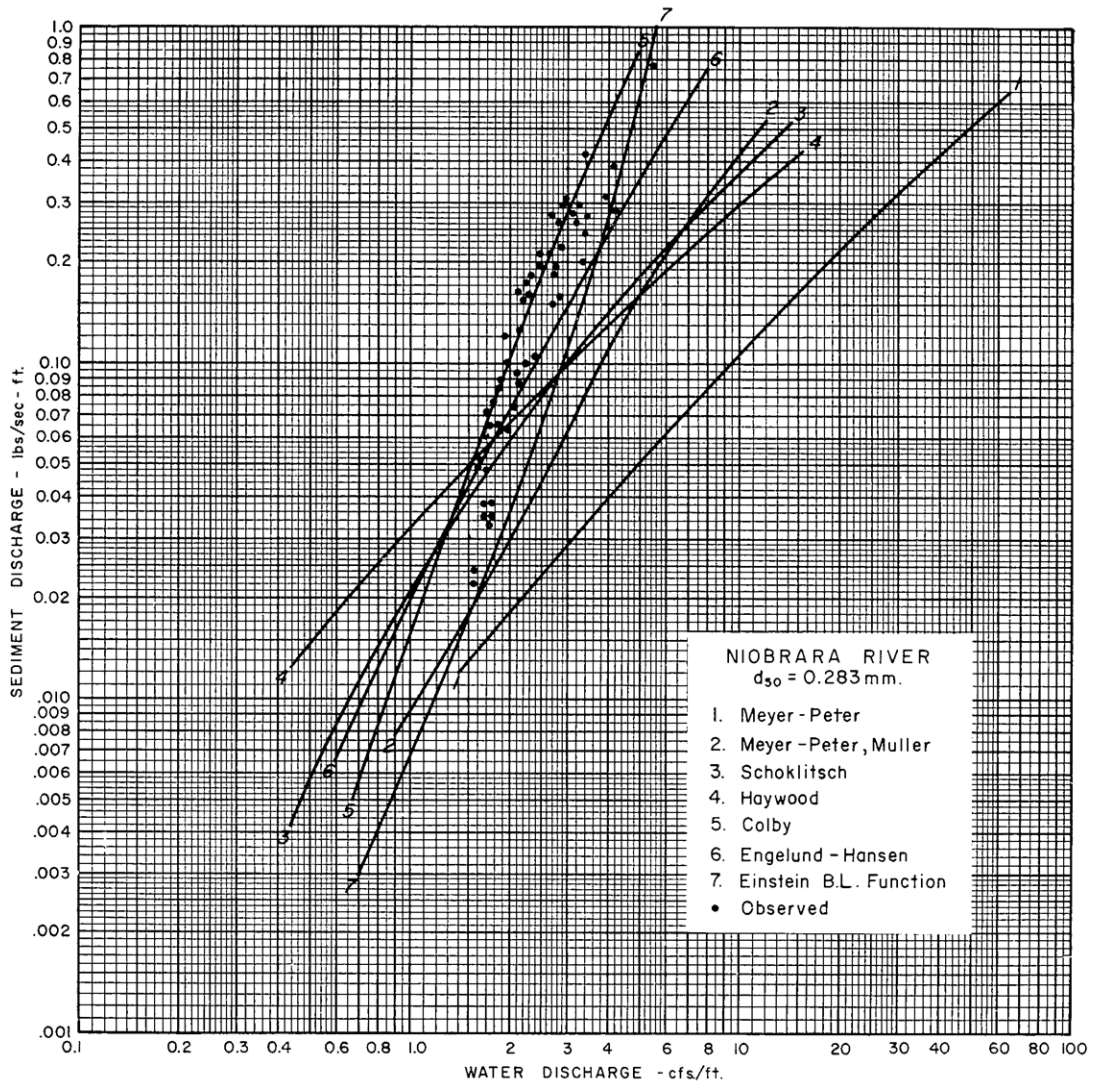


Figure 14. Sediment rating curves for Niobrara River near Cody, Nebr., according to several formulas compared with measurements.

Adapted from Vanoni, Brooks, and Kennedy (1961); American Society of Civil Engineers (1975, p. 221).

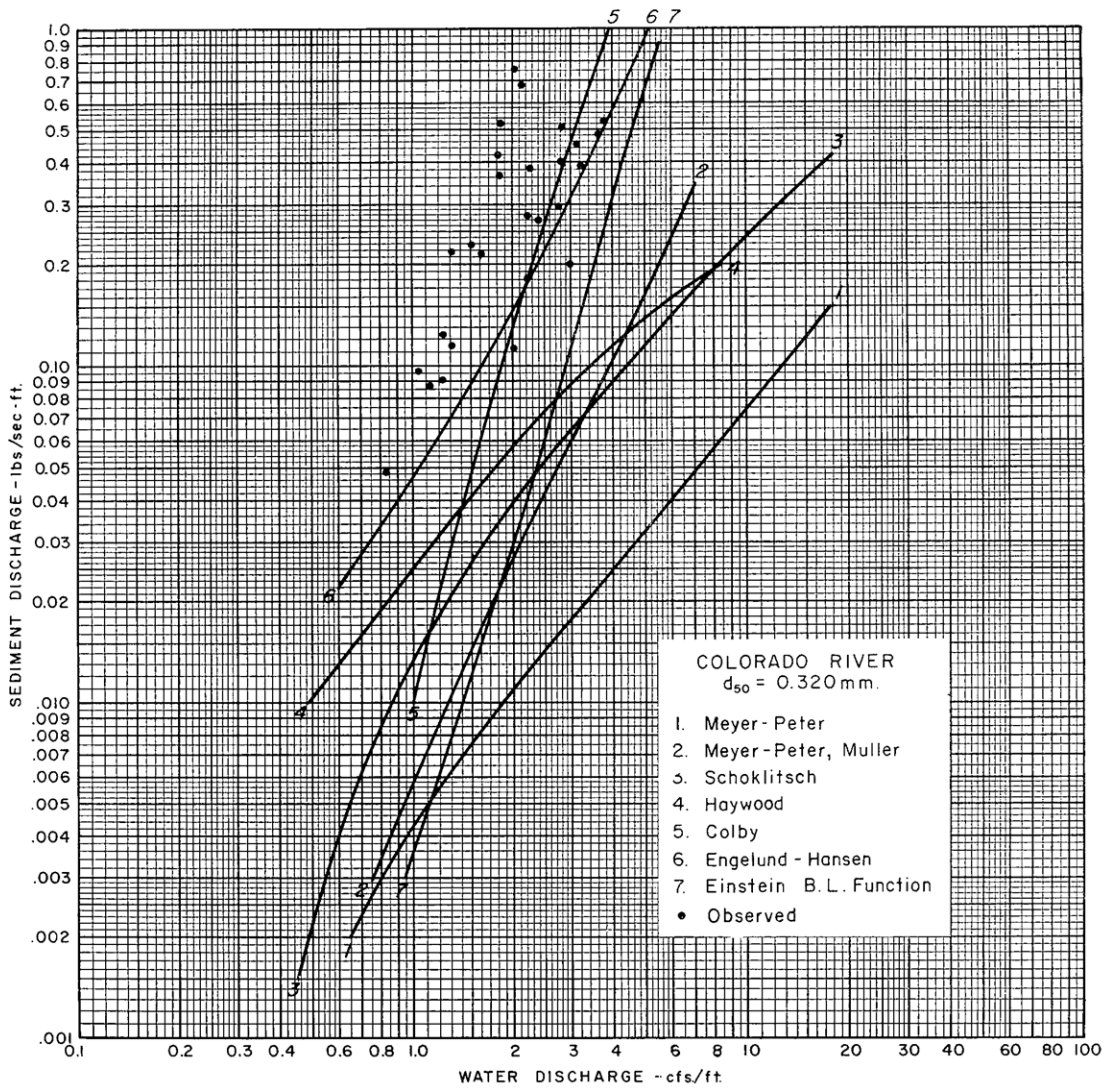


Figure 15. Sediment rating curves for Colorado River at Taylor's Ferry, AZ, according to several formulas compared with measurements.

Adapted from Vanoni, Brooks, and Kennedy (1961); American Society of Civil Engineers (1975, p. 221).



## **Glossary of Terms**

<b>Antidunes</b>	Bed forms that occur if the water velocity is higher than that forming dunes and plane beds. Antidunes commonly move upstream and are accompanied by and in phase with waves on the water surface.
<b>Armor</b>	A layer of particles, usually gravel size, that covers the bed as a coarse residue after erosion of the finer bed materials.
<b>Bed form</b>	Generic term used to describe a sand streambed. Includes ripples, dunes, plane bed, and antidunes (see figure 7).
<b>Bedload</b>	Material moving on or near the streambed by rolling, sliding, and making brief excursions into the flow a few diameters above the bed.
<b>Bed material load</b>	Portion of the sediment that is transported by a stream that contains material derived from the bed. Bed material load consists of the bedload and the portion of the suspended load that is represented in the bed sediments.
<b>Dunes</b>	Bed forms with a triangular profile having a gentle upstream slope. Dunes advance downstream as sediment moves up the upstream slope and is deposited on the steeper downstream slope. Dunes move downstream much more slowly than the stream flow.
<b>Fall diameter or standard fall diameter</b>	The diameter of a sphere that has a specific gravity of 2.65 and the same terminal velocity as a particle of any specific gravity when each is allowed to settle alone in quiescent distilled water of infinite extent and at a temperature of 24° C. A particle reaches terminal velocity when the water resistance is equal to the force of gravity.
<b>Laminar flow</b>	Low-velocity flow in which layers of fluid slip over contiguous layers without appreciable mixing.
<b>Plane bed</b>	A sedimentary bed with irregularities no larger than the maximum size of the bed material.
<b>Ripples</b>	Bed forms that have a triangular profile and are similar to dunes but much smaller. Standing waves. Water waves that are in phase with antidunes.
<b>Suspended load</b>	The part of the total sediment load that moves above the bed layer. The weight of suspended particles is continuously supported by the fluid (see wash load).
<b>Turbidity</b>	Cloudiness or haziness condition of water, which may be correlated to sediment concentration or other coloring agents.
<b>Turbulent flow</b>	A state of flow in which the fluid is agitated by crosscurrents and eddies.
<b>Uniform flow</b>	A flow in which the velocity is the same in both magnitude and direction from point to point along a reach.

**Wash load**

The part of the sediment load of a stream composed of fine particles (usually smaller than 0.062 mm) found only in relatively small quantities in the streambed. Almost all the wash load is carried in nearly permanent suspension, and its magnitude depends primarily on the amount of fine material available to the stream from sources other than the bed.

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## Symbols

Symbol	Description	Unit
A	Area of flow, cross section	Square feet
b	Channel width or water surface width	Feet
D	Depth of flow	Feet
$d_{50}$	Median size of sediment (lettered with numerical subscript denotes particle size in sediment for which the percentage by weight corresponding to subscript is finer, e.g., $d_{84}$ is size for which 84 percent of sediment by weight is finer).	Millimeters, inches, or feet
dm	Effective diameter of the bed material	Feet or millimeters
$d_s$	Particle size (unspecified)	Millimeters, inches, or feet
$F_n$ or $F_d$	Froude number; equal to $\frac{U}{(gD)^{1/2}}$	Dimensionless
f	Darcy-Weisbach friction coefficient $\frac{8gRS}{U^2}$	Dimensionless
g	Acceleration due to gravity, 32.2  or 9.8	Feet per second per second or Meters/second/second
$k_s$	Representative grain size	Feet
Q	Water discharge	Cubic feet per second
$Q_b$	Bedload discharge	Tons per day or pounds per second
$Q_s$	Water discharge effective in transporting bedload	Cubic feet per second
$Q_T$	Total bed-material discharge	Tons per day or pounds per second
q	Unit water discharge	Cubic feet per second per foot of channel width
$q_0$ or $Q_c$	Unit water discharge just sufficient to move bed material	Cubic feet per second per foot channel width
$q_b$	Unit bedload discharge	Tons per day per foot or pounds per second per foot of channel width
$q_T$	Unit bed-material discharge	Tons per day per foot or pounds per second per foot of channel width
R	Hydraulic radius	Feet
$R_b$	Hydraulic radius with respect to the bed	Feet
$R_N$ or $R_e$	Reynolds number; equal to $\frac{UD}{\nu}$ or $\frac{4UR}{\nu}$	Dimensionless
$R_*$	Boundary Reynolds number; equal to $\frac{U_* d_s}{\nu}$	Dimensionless
$R'$	Hydraulic radius with respect to the grain	Feet



Symbol	Description	Unit
$R''$	Hydraulic radius with respect to dunes and bars	Feet
$S$	Slope	Feet per foot
$S_w$	Water surface slope or hydraulic gradient	Feet per foot
$S_0$	Bed slope	Feet per foot
$S_e$	Energy gradient	Feet per foot
$S_s$	Specific gravity of sediment	Dimensionless
$T^{\circ}$	Water temperature	Degrees Fahrenheit or Celsius
$u_*$	Shear velocity ( $gDS_e$ ) <sup>1/2</sup>	Feet per second
$u'_*$	Shear velocity associated with grain roughness	Feet per second
$U$ or $V$	Mean velocity	Feet per second
$w$	Fall velocity of sediment particles	Feet per second
$\gamma$	Unit weight of water, 62.4 or 1.0	Pounds per cubic foot <i>or</i> Gram per cubic centimeter
$\gamma_s$	Unit weight of sediment, dry	Pounds per cubic foot
$\Delta\gamma$	Difference between specific weight of sediment and that of water	Pounds per cubic foot
$\delta$	Thickness of laminar sublayer	Feet
$\theta$	A form of the bed shear, $\tau_0$	Dimensionless
$\nu$	Kinematic viscosity	Square feet per second
$\mu$	Dynamic viscosity	Pound-seconds per square foot
$\rho$	Density of water	Slugs per cubic foot
$\rho_s$	Density of sediment	Slugs per cubic foot
$\psi$	A parameter indicating the ability of a flow to dislodge a given particle size (Einstein)	Dimensionless
$\phi$	A parameter describing the intensity of transport of bed material in a given size range (Einstein)	Dimensionless
$\tau_0$	Total bed shear stress	Pounds per square foot
$\tau_c$	Critical tractive stress associated with the beginning of bed movement (Shields)	Pounds per square foot
$\tau'$	Shear stress associated with grain resistance	Pounds per square foot
$\tau''$	Shear stress associated with irregularities in the bed and banks	Pounds per square foot